UASA of complex models: Coping with dynamic and static inputs

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In many fields, complex systems are modelled by a set of partial differential equations with initial and boundary conditions. For instance, in mechanics or thermodynamics, the PDEs are based on conservation laws. A particular problem is defined by a set of inputs that characterized the system of interest, embedding the initial and boundary conditions. Then, numerical methods are employed to solve the problem. In practice, the system is not accurately defined due to the uncertainty about some inputs. Uncertainty and sensitivity analyses (UASA) can help assess the impact of this lack of knowledge onto the model responses ([1,2]). Let $\boldsymbol{y} = g(\boldsymbol{\omega^d}(x,\theta), \boldsymbol{\omega^s}(\theta), x)$ be the response of interest where: $x \in \mathcal{D}$ is the spatial/time variable, $\boldsymbol{\omega^d}$ is a set of random fields (dynamic inputs) and $\boldsymbol{\omega^s}$ is a set of random variables (static inputs) (see figure 1). As an example, in building energy modelling, $\boldsymbol{\omega^s}$ embeds the thermophysical properties of the materials used in the building while $\boldsymbol{\omega^d}$ represents the weather data.
In this communication, we address the issue of performing UASA with these two kinds of uncertain inputs. Indeed, in the literature, such an issue is rarely addressed (except, for instance, in [3]).

For the sake of simplicity, we assume that random variables are independent and defined by their marginal distribution. The random fields are also assumed independent and normally distributed with mean $\bar{\omega}_i(x)$ and covariance function $C_i(x_1, x_2)$, $i = 1, \cdots, N_d$, N_d denoting the number of dynamic inputs. Monte Carlo based methods can be used to perform UASA of such a model. But, while generating static inputs samples is not an issue, it is not straightforward to generate samples that satisfy the desired random fields distribution. One possibility is to resort to the truncated Karhunen-Loeve (KL) expansion. The former expands a random field as follows:

$$\omega_i^d(x,\theta) \simeq \bar{\omega}_i^d(x) + \sum_{k=1}^{M_i} \sqrt{\lambda_{ki}}\xi_{ki}(\theta) f_{ki}(\theta), \tag{1}$$

where λ_{ki} and f_{ki} are the deterministic eigenvalues and eigenfunctions of the covariance function $C_i(x_1, x_2)$, $\boldsymbol{\xi_i}(\theta)$ is a set of independent standard normal variables and M_i is the number of KL-terms. The eigenmodes depend on the choice of the covariance function and are determined by solving the Fredholm integral equation of the second kind given by:

$$\int_{\mathcal{D}} C_i(x_1, x_2) f_{ki}(x_1) dx_1 = \lambda_{ki} f_{ki}(x_2). \tag{2}$$

Equation (2) can be solved using a wavelet-Galerkin scheme ([4]). The advantage of this approach is to avoid tedious quadratures by using wavelet transform, alleviating computational effort.
In practice, we retain the first M_i eigenmodes that contain the 95% of the variance of the input ω_i^d. The number of eigenmodes retained depends on the choice of the covariance function and may be very different from one input to another. Note that once the eigenmodes are obtained for all the dynamic inputs, UASA of the model output are performed through the random vectors $\{\underbrace{\boldsymbol{\xi_1}, \cdots, \boldsymbol{\xi_{N_d}}}_{\boldsymbol{\omega^d}}, \boldsymbol{\omega^s}\}$. Consequently, the effect of the group of factors $\boldsymbol{\xi_i}$ is the one of the dynamic input ω_i^d. This effect can be estimated with sampling-based methods such as Sobol' method ([5]).

The approach is applied to a building energy model. This model presents dynamic inputs as dry bulb temperature, direct and diffuse radiations, humidity, speed and direction of wind, and static inputs as the thermal properties of the materials. The model response of interest is the energy consumption (scalar output).

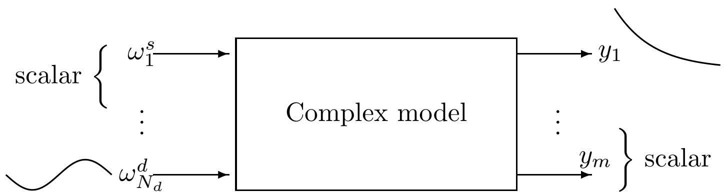

Fig. 1 - Complex model with static and dynamic inputs

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